

Solving Rational Expressions Notes

Solving an Equation Containing Rational Expressions

1. Multiply both sides of the equation by the LCD.
2. Solve the resulting equation.
3. Check the solution to determine whether it is an excluded value and therefore extraneous.

Sample Problem 1: Solve the following rational expression by finding the value of the unknown variable.

1. $\frac{3}{x-7} + 5 = \frac{8}{x-7}$

Solution:

$$\frac{3}{x-7} + 5 = \frac{8}{x-7}$$

$$x-7 \left[\frac{3}{x-7} + 5 = \frac{8}{x-7} \right]$$

$$\cancel{(x-7)} \left(\frac{3}{\cancel{x-7}} \right) + (x-7)5 = \cancel{(x-7)} \left(\frac{8}{\cancel{x-7}} \right)$$

$$3 + 5(x-7) = 8$$

$$3 + 5x - 35 = 8$$

$$5x = 8 + 32$$

$$5x = 40$$

$$x = 8$$

2. $\frac{v}{v-3} = 4 - \frac{3}{3-v}$

Solution:

$$\frac{v}{v-3} = 4 - \frac{3}{3-v}$$

$$v-3 \left[\frac{v}{v-3} = 4 - \frac{3}{3-v} \right]$$

$$\cancel{(v-3)} \left(\frac{v}{\cancel{v-3}} \right) = (v-3)(4) + \cancel{(v-3)} \left(\frac{3}{\cancel{v-3}} \right)$$

$$v = 4v - 12 + 3$$

$$-3v = -9$$

$$v = 3$$

3. $\frac{x}{2} + \frac{x}{3} = 5 + x$

Solution:

$$\frac{x}{2} + \frac{x}{3} = 5 + x$$

$$6 \left[\frac{x}{2} + \frac{x}{3} = 5 + x \right]$$

$$6 \left(\frac{x}{2} \right) + 6 \left(\frac{x}{3} \right) = 6(5) + 6(x)$$

$$3x + 2x = 30 + 6x$$

$$-30 = 6x - 5x$$

$$x = -30$$

4. $\frac{y-9}{y} = \frac{3}{4}$

Solution:

$$\frac{y-9}{y} = \frac{3}{4}$$

$$4y \left[\frac{y-9}{y} = \frac{3}{4} \right]$$

$$4y \left(\frac{y-9}{y} \right) = 4y \left(\frac{3}{4} \right)$$

$$4y - 36 = 3y$$

$$y = 36$$

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Sample Problem 2: Find solution for the following rational expressions.

5. $\frac{m}{m-2} = \frac{m}{m-2} + 7$

Solution:

$$m-2 \left[\frac{m}{m-2} = \frac{m}{m-2} + 7 \right]$$

$$\cancel{m-2} \left(\frac{\cancel{m}}{\cancel{m-2}} \right) = \cancel{m-2} \left(\frac{\cancel{m}}{\cancel{m-2}} \right) + (m-2)7$$

$$m = m + 7m - 14$$

$$7m = 14$$

$$m = 2$$

Checking:

$$\frac{m}{m-2} = \frac{m}{m-2} + 7$$

$$\frac{2}{2-2} = \frac{2}{2-2} + 7$$

$$\frac{2}{0} = \frac{2}{0} + 7$$

No solution

The solution set is $\{\emptyset\}$

6. $1 + \frac{6}{x^2-9} = \frac{1}{x-3}$

Solution:

$$1 + \frac{6}{x^2-9} = \frac{1}{x-3}$$

$$(x+3)(x-3) \left[1 + \frac{6}{x^2-9} = \frac{1}{x-3} \right]$$

$$(x+3)(x-3)(1) + \cancel{(x+3)(x-3)} \left(\frac{6}{\cancel{x^2-9}} \right) = (x+3) \cancel{(x-3)} \left(\frac{1}{\cancel{x-3}} \right)$$

$$(x+3)(x-3) + 6 = (x+3)$$

$$x^2 - 9 + 6 = x + 3$$

$$x^2 - 3 = x + 3$$

$$x^2 - x - 3 - 3 = 0$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x-3=0; x+2=0$$

$$x=3; x=-2$$

Checking:

$$x=3$$

$$1 + \frac{6}{(3)^2-9} = \frac{1}{3-3}$$

$$1 + \frac{6}{0} = \frac{1}{0}$$

3 is not a solution.

$$x=-2$$

$$1 + \frac{6}{(-2)^2-9} = \frac{1}{-2-3}$$

$$1 - \frac{6}{5} = -\frac{1}{5}$$

$$\frac{-5+6}{5} = -\frac{1}{5}$$

$$-\frac{1}{5} = -\frac{1}{5}$$

The solution is -2